CEA-EDF-Inria Summer School on Numerical Analysis

Numerical methods for interface problems in fluids and solids with discontinuities

June 23rd – July 4th, 2013, Cadarache, France

Aim:

Long-established numerical techniques, as the finite element method, are well-suited for the approximation of smooth fields. In practice, however, many applications require solutions with localized non-smooth features (discontinuities, singularities, etc.). The modeling of fractures in materials, or of multi-phase flow are typical examples. The aim of the summer school is to give a thorough introduction to the mathematical models and numerical techniques used for the efficient simulation of this type of problems. The school is intended for researchers, engineers and PhD students.

Organizing committee: M.A. Fernández (Inria), P. Massin (EDF), J. Segré (CEA)

Lectures:

- **Lecture 1: Interface description, methods for enforcing interface constraints**
  
  **John Dolbow (Duke University, USA)**

  The course will focus on methods to represent evolving discontinuities and interfaces, with particular attention placed on the level-set approach. The level-set method will be introduced and compared to existing techniques, such as volume-of-fluid methods. Then a recent addition, called the gradient-augmented level-set method, will be introduced and presented for multi-phase flow. The second half of the course will focus on methods for imposing constraints on embedded interfaces. We will begin by reviewing classical penalty methods, and demonstrate how these techniques can fail for embedded interface problems. We will then focus on the development of both stable and stabilized methods for enforcing constraints on embedded interfaces. The method of vital vertices will be discussed, as well as a weighted Nitsche approach. Finally, applications to frictional contact and higher-order problems will be presented.

- **Lecture 2: Moving interfaces in solids**

  **Nicolas Moës (École Centrale de Nantes, France)**

  The course will focus on the Extended Finite Element method, associated to the notion of level set developed in the previous set of lectures by John Dolbow. Application to fracture mechanics will be discussed. To deal with crack initiation the concept of Thick Level Set will then be introduced. While standard X-FEM deals with pre-existing cracks and their propagation, the TLS model is a new theoretical model
allowing to initiate cracks or to model complex cracking patterns (branching and merging). The last domain investigated will be the one of variational inequalities associated to domain evolutions which has to be taken into account in case of plasticity or contact mechanics, for example. Again, level set are used and the approach is coined Inequality Level Set.

• Lecture 3: Interface capturing methods for multi-phase fluid mechanics
  \textit{Samuel Kokh (Maison de la Simulation, France)}
  \textit{Frédéric Lagoutière (Unviersité Paris-Sud, France)}

  The purpose of this lectures series is to present simulation methods for evolving sharp interfaces that separate compressible components. These methods are based on two key ingredients: a coupling procedure based on an augmented model and anti-diffusive transport numerical schemes that enable the evolution of discretized characteristic functions with very low numerical diffusion. We will show how these features can be articulated to elaborate robust solution strategies for the simulation of flows with two or more components, while ensuring conservation properties for partial mass, mass, energy and momentum.

\textbf{Schedule:}

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Detailed Program (ongoing):

- **Lecture 1:** Interface description, methods for enforcing interface constraints

**Lectures (6h x 1h30)**

**LS I**
- Theory of curve and surface evolution
- Standard techniques for representing interfaces

**LS II**
- Level-set and fast marching methods
- Stabilized discretizations based on finite elements

**LS III**
- The gradient-augmented level-set approach
- Redistancing and narrow-banding
- Applications to multi-phase flow

**Constraints I**
- Classical penalty and augmented lagrange approaches to enforcing constraints
- Introduction to Nitsche’s method

**Constraints II**
- Methods based on lagrange multipliers with vital vertices
- Weighted Nitsche approach
- Conditioning issues

**Constraints III**
- Application to frictional contact problems
- Application to higher-order problems with splines

**Hands-on (6 x 1h30)**

- LS I: constructing interfaces using Young’s algorithm
- LS II: level-sets for etching simulations
- LS III: fluid-flow simulations
- Constraints I: Derivation of Nitsche’s method
- Constraints II: Building vital-vertex graph
- Constraints III: Numerical analysis exercises
• Lecture 2: Fracture mechanics of solids

Lectures (6h x 1h30)

X-FEM I
- Motivations for the eXtended Finite Element Method (X-FEM)
- Discontinuity in a field, it's derivative or in matter

X-FEM II
- Modeling cracks with the X-FEM
- Examples of application in fracture mechanics and material modeling.

TLS I
- The importance of a characteristic length in damage mechanics, the issue of spurious localization
- Classical non-local approaches for damage

TLS II
- Fundamentals of Thick Level Set (TLS) model to go from damage to fracture
- Examples of simulations in quasi-static and dynamics for quasi-brittle media

ILS I
- Examples of variational inequalities in mechanics inextensibility, contact, plasticity
- Difficulties with traditional algorithms
- Fundamentals of the ILS approach (Inequality Level Set).

ILS II
- Application to inextensibility material constraint
- Application to contact and adhesion

Hands-on (6 x 1h30)
- X-FEM I: Full resolution by hand of a 1D bar with different materials and a cohesive interface dealt with X-FEM.
- X-FEM II: a) Demonstrate the equivalence of Heaviside enrichment and Hansbo's enrichment. b) X-FEM for dynamics, critical time step. c) The issue with naïve Lagrange multiplier on a 2 element case.
- TLS I: 1D comparison between TLS model and cohesive model
- TLS II: Fiber pull-out and discussion of the different regimes in the solution.
- ILS I: Resolution of a 1D bar with a limit of extensibility material constraint.
- ILS II: Resolution of a membrane contacting the ground (involving superficial tension or not).
• **Lecture 3: Interface capturing methods for multi-phase fluid mechanics**

**Lectures (6h x 1h30)**

I
- Anti-diffusive discretization for the transport of an interface in 1D
- Anti-diffusive discretization for the transport of N interfaces in 1D
- Extension to the multi-dimensional case for cartesian grids (directional splitting)

II: Anti-diffusive discretization for the transport of an interface on an unstructured grid

III:
- Interface capture technique for the simulation of a compressible flow involving two compressible materials separated by an interface
  - Coupling by means of an augmented model
  - Properties of some augmented models
- Interface capture technique for the simulation of a compressible flow involving an arbitrary number of compressible materials separated by an interface

IV:
- Anti-diffusive capture scheme for two-material interface flows on cartesian grids
- Anti-diffusive capture scheme for N-material interface flows on cartesian grids

V:
- Anti-diffusive capture scheme for two-material interface flows on cartesian grids
- Anti-diffusive capture scheme for N-material interface flows on cartesian grids

VI:
- A connection between the Lagrange-Projection strategy and an operator splitting approach
- Vofire strategy for a two-component model on unstructured grid
- Anti-diffusive mechanism: towards reconstruction schemes for the gas dynamics

**Hands-on (6 x 1h30)**
- Vofire scheme for 2D triangular discretization
- ...